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T.R.E. REPORT

ATI No. 1006

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SECRET

V-7598-C

T.R.E. REPORT T1880.

SECRET (ELECTRICAL SIMULATOR AND PROPOSED CONTROL SYSTEM FOR LONG SHOT)

SUMMARY

Long Shot is the code name for an air-launched rocket. The weapon is designed primarily to be launched from a fighter aeroplane and to be automatically guided towards an enemy aircraft. The type of control described in this paper is designed for a line of sight trajectory in which the rocket is controlled to fly down the axis of a radio beam which is kept pointed at the target. In order to investigate the stability and performance of various types of control system an electrical simulator was built, the function of the simulator being to produce voltages proportional to the movement of the rocket after the application of the controlling rudders. The aerodynamic characteristics of the rocket and the way in which they are simulated are described.

A control system has been evolved which gives a satisfactory performance on the simulator. A theoretical description of this system is given and its performance on the simulator is also described.

INTRODUCTION

Before describing the simulator it is necessary to describe the aerodynamic characteristics of the rocket which it is required to simulate.

This is dealt with in Section 1, for which the information has been supplied by R.A.E. Section 2 describes the simulator designed for these aerodynamic characteristics. Section 3 gives a theoretical description of the type of control system which is at present proposed for controlling the rocket. Section 4 gives an account of the results of the initial tests of this control system using the simulator described in Section 2. Appendix I gives a list of the symbols used in describing the aerodynamic characteristics of the rocket.

1. AERODYNAMIC CHARACTERISTICS OF THE ROCKET

It is proposed that the rocket will only be controlled after the propulsion has ceased, the duration of the burning being of the order of 1 to $1\frac{1}{2}$ seconds. Thus during the control period the velocity will be approximately constant and will be of the order of 1500 ft. per second. In fig. (i) PA is a fixed datum line, PB is the line of sight, R is the position of the rocket, RE is its heading and RF is the direction in which the centre of gravity of the rocket is moving. CD is a line through R parallel to the datum line PA. Let DRF = ψ, DRE = ϕ and ERF = α. Let the lateral distance of R from the datum be h_o and from the line of sight be h. Let $h_1 = h_o - h$ be the lateral distance of the line of sight from the datum. Let V be the velocity of the centre of gravity of the rocket. Then assuming that α, ϕ and ψ are small angles the following relations apply:-

$$\alpha = \phi - \psi \qquad 1.1$$

$$h = h_o - h_1 \qquad 1.2$$

$$\frac{dh_o}{dt} = V \sin\psi = V\psi \qquad 1.3$$

Now the lift on the wings will be proportional to the angle of incidence α, and hence the lateral acceleration $\frac{d^2h_o}{dt^2}$ will be proportional to α, this may conveniently be written in the form

$$\alpha = \frac{K_1}{V}\frac{d^2h_o}{dt^2} \qquad 1.4$$

where K_1 is a constant with the dimensions of time.

From 1.3 and 1.4 we have

$$\alpha = K_1 \quad \frac{d\psi}{dt} \qquad 1.5$$

/and

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and from 1.1 and 1.5 $\phi = \psi + K_1 \frac{d\psi}{dt}$ --- 1.6

It will be seen that this equation defines the relation between the heading of the rocket and the path of the C.G., i.e. it defines the amount of yaw.

It is proposed that the wing system will be designed so that during the control period the rocket has neutral weathercock stability. The controls will consist of small pairs of 'bang-bang' rudders, i.e. the rudders will be either hard over one way or hard over the other. Thus the effect of the rudders will be to apply constant torque to the rocket about its centre of gravity which will in turn produce a constant angular acceleration of the rocket heading. Thus if this acceleration is Q radians/sec^2 we have

$$\frac{d^2\phi}{dt^2} = \pm Q \qquad \text{--- 1.7}$$

Differentiating 1.6 twice and substituting in 1.7 we have

$$\frac{d^2\psi}{dt^2} + K_1 \frac{d^3\psi}{dt^3} = \pm Q \qquad \text{--- 1.8}$$

or from 1.3 $\frac{d^3h_o}{dt^3} + K_1 \frac{d^4h_o}{dt^4} = \pm QV = \pm P$ --- 1.9

where $P = QV$ is a constant with dimensions of ft/sec^3.

EFFECT OF WEATHERCOCK STABILITY AND ROTARY DAMPING

Suppose the centre of pressure of the wings and body is not exactly coincident with the centre of gravity of the rocket. Then the lift force will apply a torque about the C.G. which may oppose or assist the torque due to the rudders, depending on whether the C.P. is behind or in front of the C.G., i.e. whether the weathercock stability is positive or negative. Since from 1.5 the lift is proportional to $\frac{d\psi}{dt}$, equation 1.7 will now become

$$\frac{d^2\phi}{dt^2} = \pm Q - K_2 \frac{d\psi}{dt} \qquad \text{--- 1.7a}$$

where K_2 is a constant with dimensions of (time)$^{-1}$ whose value depends on the distance between the C.G. and the C.P., being positive if the C.P. is behind the C.G. and negative if it is in front.

The effect of rotary damping is to apply a torque about the C.G. of the rocket proportional to the angular velocity of the rotation about the C.G., i.e. proportional to $\frac{d\phi}{dt}$. This torque will oppose the torque due to the rudders. Thus equation 1.7 becomes

$$\frac{d^2\phi}{dt^2} = \pm Q - K_3 \frac{d\phi}{dt} \qquad \text{--- 1.7b}$$

where K_3 is a constant with dimensions of (time)$^{-1}$. In the presence of both rotary damping and weathercock stability equation 1.7 becomes

$$\frac{d^2\phi}{dt^2} = \pm Q - K_3 \frac{d\phi}{dt} - K_2 \frac{d\psi}{dt} \qquad \text{--- 1.7c}$$

If we substitute for ϕ from 1.6 we have

$$\frac{d^2\psi}{dt^2} + K_1 \frac{d^3\psi}{dt^3} = \pm Q - K_3 \left(\frac{d\psi}{dt} + K_1 \frac{d^2\psi}{dt^2}\right) - K_2 \frac{d\psi}{dt}$$

$$(K_2 + K_3) \frac{d\psi}{dt} + (1 + K_1K_3) \frac{d^2\psi}{dt^2} + K_1 \frac{d^3\psi}{dt^3} = \pm Q \qquad \text{--- 1.10}$$

/or from

or from 1.3 $$(K_2 + K_3)\frac{d^2h_o}{dt^2} + (1 + K_1K_3)\frac{d^3h_o}{dt^3} + K_1\frac{d^4h_o}{dt^4} = \pm P \quad \text{--- } 1.11$$

For the design of the simulator the following values were assumed for constants, these values having been calculated by RAE for the proposed design of Long Shot.

$Q = 7$ radians/sec^2 $\quad V = 1500$ ft/sec $\quad K_1 = \frac{1}{4}$ second.

These figures are only tentative at present, in particular consideration is being given to the use of rudders giving a larger value of angular acceleration Q.
However they will be used in this report to give an indication of the orders of magnitude involved in the design of the simulator and control system.

In order to investigate the tolerance allowed on the distance between the C.P. and C.G., the constant K_2 has been varied between 0 and 60 sec^{-1}. RAE have calculated that the constant K_3 can be neglected in comparison with K_1 and K_2.

2. DESIGN OF SIMULATOR

The purpose of the simulator is to reproduce electrically the motion of the rocket when the rudders are applied. That is to produce a voltage proportional to h_o, whose derivatives obey equation 1.9 in the simplest case, or equation 1.11 in the more complicated case. The output can then be applied through the proposed control system to actuate a relay representing the rudders.

A block schematic of the simulator and the way it is used to test a control system is shown in fig.(ii). The relay actuated by the control system carries a single change-over contact. The fixed contacts are taken to steady voltages + V and -V so that the moving contact is connected to +V when the relay is energised and -V when it is not energised. This voltage represents the torque applied by the rudders i.e. $V = a_oQ$ where a_o is a constant.

Let us assume first the simple case with neutral weathercock stability and negligible rotary damping. Then from 1.7 we have

$$\frac{d^2\phi}{dt^2} = \pm Q$$

$$\text{and hence } V_o = a_o\frac{d^2\phi}{dt^2} \qquad \text{--- } 2.1$$

Now if we apply V_o to the input of an integrator, then the output voltage V_1 will be given by $V_1 = a_1\dfrac{d\phi}{dt}$ --- 2.2

This voltage V_1 is then put through a circuit representing the incidence lag which produces an output voltage given by

$$V_2 + K_1\frac{dV_2}{dt} = V_1$$

where the time constant K_1 is made equal to that defined by equation 1.6.

$$\text{Thus} \quad V_2 + K_1\frac{dV_2}{dt} = a_1\frac{d\phi}{dt} \qquad \text{--- } 2.3$$

Differentiating equation 1.6 we have

$$\frac{d\psi}{dt} + K_1\frac{d^2\psi}{dt^2} = \frac{d\phi}{dt}$$

/and

and comparing this with equation 2.3 we see that

$$V_2 = a_1 \frac{d\psi}{dt} \qquad --- \quad 2.4$$

or since

$$\psi = \frac{1}{V}\frac{dho}{dt}$$

$$V_2 = \frac{a_1}{V}\frac{d^2ho}{dt^2}$$

or

$$V_2 = a_2 \frac{d^2ho}{dt^2} \qquad --- \quad 2.5$$

where $a_2 = \frac{a_1}{v}$

The voltage V_2 is then applied to a second integrator whose output voltage V_3 is therefore given by

$$V_3 = a_3 \frac{dho}{dt} \qquad --- \quad 2.6$$

A third integrator is then used to integrate V_3 to produce

$$V_4 = a_4 ho \qquad --- \quad 2.7$$

which represents the position of the rocket from a datum.
A voltage V_5 is then connected in series with V_4 to represent the position of the line of sight from the datum, i.e.

$$V_5 = a_4 h_1$$

So that the final output $V_6 = V_4 - V_5$

$$V_6 = a_4 (h_o - h_1)$$

$$V_6 = a_4 h \qquad 2.8$$

Thus V_6 represents the error from the line of sight.

The voltage V_5 can be varied in any desired manner to represent movement of the line of sight and the response of the control system in reducing the error V_6 can be studied.

The time and voltage scales of the simulator can of course be adjusted as required by using suitable values for the dimensional constants a_o, a_1, a_2, etc. Simulators for Long Shot have been constructed with both 20 to 1 and 1 to 1 time scales. The former has the advantage that it is easier to observe and record the results, since normal recording voltmeters with time constants of the order of a quarter of a second may be used. The latter is necessary in order to test the final control circuits and investigate the effects of lags in relays etc. which cannot easily be scaled down. An oscillograph has been used for recording the results of the 1/1 simulator. To illustrate the method of designing the simulator the circuit of the 1/1 time scale version will be described. The first version of this simulator used valve integrators throughout, the essential details of the circuit being shown in fig. (iii)(a). For simplicity the screen and grid bias supplies of the valves have been omitted.

The integrators Y_1, Y_3 and Y_4 use the normal Miller circuit. Provided that the gain of the valve is large and that the input voltage is large compared with the excursions of the grid then we can easily show that the anode voltage will be the integral of the input voltage. Thus for Y_1, assuming the above conditions we have

$$\frac{V_o}{R_1} + C_1 \frac{dV_a}{dt} = 0$$

/where

where Va is the anode voltage

$$\text{or } Va = \frac{-1}{C_1R_1} \int V_o \, dt$$

For convenience in obtaining the required D.C. level at the grid of the next valve the output is taken from the centre of a potentiometer connected between the anode of the valve and the -150 volt line. If this output voltage is V_1 it is clear that $\frac{dV_1}{dt} = \frac{1}{2}\frac{dVa}{dt}$ and hence.

$$\frac{V_o}{R_1} + 2\,C_1\frac{dV_1}{dt} = o$$

$$V_1 = -\frac{1}{2C_1R_1}\int V_o \, dt$$

The relay contacts are taken to $\pm$ 60 volts and since $V_o = \pm\, a_oQ$ where $Q = 7$ rad/sec^2, therefore

$$a_o = \frac{60}{7}\ \frac{\text{volts}}{\text{rad/sec}^2}$$

$$\text{Now } V_1 = -\frac{1}{2C_1R_1}\int\frac{a_o d^2\phi}{dt^2} \text{ since } Vo = a_o\frac{d^2\phi}{dt^2}$$

$$\text{Hence } V_1 = -\frac{1}{2C_1R_1}\ \frac{60}{7}\frac{d\phi}{dt} = a_1\frac{d\phi}{dt} \qquad --- \quad 2.9$$

It is assumed that the system is at rest at the start of an experiment so that the constants of integration will be zero. The incidence lag is represented simply by the resistance R_2 and condenser C_2. Thus if V_2 is the voltage at the grid of Y_2 we have

$$\frac{V_1 - V_2}{R_2} = C_2\ \frac{dV_2}{dt}$$

$$V_1 = V_2 + C_2R_2\frac{dV_2}{dt}$$

Thus in order to simulate an incidence lag of $\frac{1}{4}$ sec. it is necessary to make $C_2R_2 = \frac{1}{4}$ sec. Then if $V_1 = a_1\frac{d\phi}{dt}$, $V_2 = a_1\frac{d\Psi}{dt}$ or $V_2 = a_2\frac{d^2h_o}{dt^2}$ where

$$a_1 = a_2 V$$

$$a_1 = 1500\ a_2$$

Now the first valve will only operate correctly over a range of anode voltage from about 30 to 270 volts, which corresponds to a range of $\pm$ 60 volts of V_1. Thus the constant a_1 must be such that V_1 never exceeds 60 volts. We shall see later that a maximum value attained by $\frac{d^2h_o}{dt^2}$ is about 10g i.e. 320 ft/sec^2, but that $\frac{d\phi}{dt}$ may momentarily attain values of three to four times the maximum value of $\frac{d\Psi}{dt}$. Thus a suitable scale for V_2 is to make 10 volts represent 10g, so that the maximum value attained by V_1 will be 30 to 40 volts, which is well within the capabilities of the first valve.

$$\text{Thus we make } a_2 = \frac{10}{320}\ \frac{\text{volts}}{\text{ft/sec}^2} = .0313\ \frac{\text{volts}}{\text{ft/sec}^2}$$

/and

and hence $a_1 = \frac{1500}{32} \frac{\text{volts}}{\text{Rad/sec}} = 46.8 \frac{\text{volts}}{\text{Rad/sec.}}$

We can now calculate from 2.9 the constant C_1R_1 for the first integrator

$$\text{since } a_1 = -\frac{60}{14C_1R_1}$$

the negative sign in equation 2.9 merely indicates the normal 'reversing' effect of a valve and means that the sign conventions for V_o and V_1 will be reversed.

$$\text{Thus } C_1R_1 = \frac{60}{14} \times \frac{32}{1500}$$

$$C_1R_1 = .091 \text{ seconds}$$

Thus suitable values are R_1 = 910,000 ohms C_1 = 0.1/uf.

The valve Y_2 is simply a cathode follower to prevent the later stages loading the condenser C_2.

The equation for V_3 will be

$$\frac{V_2}{R_3} = 2C_3 \frac{dV_3}{dt}$$

$$\text{thus } V_3 = \frac{1}{2C_3R_3} \int V_2 dt$$

$$\text{and since } V_2 = a_2 \frac{d^2h_o}{dt^2}$$

$$V_3 = \frac{a_2}{2C_3R_3} \frac{dh_o}{dt}$$

$$V_3 = a_3 \frac{dh_o}{dt}$$

Again V_3 must never exceed $\pm$ 60 volts and since we shall be dealing with velocities up to 300 to 400 ft/sec., it will be safe to make 300 ft/sec correspond to 30 volts, i.e. $a_3 = \frac{1}{10} \frac{\text{volts}}{\text{ft/sec.}}$

$$\text{Thus since } a_3 = \frac{a_2}{2C_3R_3}$$

$$\therefore \quad C_3R_3 = \frac{a_2}{2a_3}$$

$$= \frac{10}{320} \times \frac{10}{2}$$

$$C_3R_3 = .156 \text{ seconds}$$

Suitable values for C_3 and R_3 are 0.1/uf and 1.56 Meg. ohms.

/Finally

Finally for V_4 we have

$$\frac{V_3}{R_4} = 2C_4\frac{dV_4}{dt}$$

$$V_4 = \frac{1}{2C_4R_4}\int V_3 dt$$

and since $V_3 = a_3 \frac{dh_o}{dt}$

$$V_4 = \frac{a_3}{2C_4R_4} h_o$$

It is expected that the output of the radio receiver in the rocket will be at a level of about $\frac{1}{15}$ volt/ft. and hence it is convenient to make

$$a_4 = \frac{a_3}{2C_4R_4} = \frac{1}{15} \text{ volt/ft.}$$

$$\text{thus } C_4R_4 = \frac{a_3 \times 15}{2}$$

$$= \frac{15}{20}$$

$$C_4R_4 = 0.75 \text{ seconds}$$

Convenient values are $C_4 = 0.15\mu f$ $R_4 = 5$ megohms.

The voltage V_4 is taken to the centre tap of a floating 30 volts battery and the output V_6 is taken from the slider of the potentiometer R_7 connected across the battery. The voltage between the centre tap of the battery and the slider of R_7 represents h_1, the position of the line of sight from the datum and hence $V_6 = a_4 (h_o - h_1) = a_4 h$ where h is the error from the line of sight. Thus when the relay at the input of the simulator is opened or closed, V_6 will vary in exactly the same way as the lateral error of the rocket from the line of sight will vary when the rudder is operated.

For the purpose of testing certain parts of the radio receiver for the rocket it is necessary to have the output voltage representing h_o in the form of a sine wave of variable amplitude or possibly as a sinusoidal modulation of the amplitude of pulses and for these purposes it is more convenient to have the output appearing as the rotation of a potentiometer. For this reason the last integrator in the simulator described above was later replaced by a velodyne electro-mechanical integrator. The essential details of this integrator are shown in fig. (iv).

The velodyne consists of a split field motor coupled to a small D.C. generator. The motor armature is constantly excited and the motor field is supplied from a high gain D.C. amplifier. The generator field is constantly excited so that the generator armature produces a voltage V_7 proportional to the speed of rotation of the motor. The voltage V_3 from the second integrator Y_3 in the simulator described above is connected through R_1 to the input of the D.C. amplifier. The generator voltage V_7 is also connected to the input of the amplifier through the resistance R_2. Now it is clear that any departures of the input of the amplifier from zero will cause a large torque to be applied to the motor and the motor will accelerate rapidly until the speed has risen to such a value that the current through R_2 due to V_7 is exactly equal and opposite to that through R_1 due to V_3 and the motor will then continue to run at that speed. Thus we see that the speed of the motor will be proportional to the voltage V_3 and hence the angular position of the output shaft will be proportional to the integral of V_3. To calculate the constant of proportionality we can assume that the input of the amplifier must remain at zero voltage within very small limits and hence

/ $\frac{V_3}{R_1}$

$$\frac{V_3}{R_1} = \frac{V_7}{R_2}$$

Now $V_7 = a_7 \frac{d\theta_1}{dt}$ where θ_1 is the angular position of the output shaft is radians. Now with the velodyne used in this case the constant a_7 is 0.267 $\frac{\text{volts}}{\text{Rad/sec}}$

$$\therefore \frac{V_3}{R_1} = \frac{.267}{R_2} \frac{d\theta_1}{dt}$$

$$\theta_1 = \frac{R_2}{.267R_1} \int V_3 dt$$

The motor is coupled through a 100/1 gear ratio to a potentiometer, so that if θ_2 is the angular position of the potentiometer.

$$\theta = 100 \ \theta_2$$

$$\text{Hence } \theta_2 = \frac{R_2}{26.7R_1} \int V_3 dt$$

Now we have arranged that $V_3 = a_3 \frac{dho}{dt}$

where $a_3 = \frac{1}{10} \frac{\text{volts}}{\text{ft/sec}}$

$$\text{Hence } \theta_2 = \frac{R_2 \ \ ho}{26.7R_1 \ 10}$$

The potentiometer had a travel of about 300° and it was decided to make this represent 900 ft., hence if

$$\theta_2 = a_8 h_o$$

$$a_8 = \frac{300}{57.3} \times \frac{1}{900} \text{ Rad/ft.}$$

$$a_8 = \frac{1}{171.9} \text{ Rad/ft.}$$

$$\text{Thus } \frac{R_2}{267R_1} = a_8 = \frac{1}{171.9}$$

$$\therefore \frac{R_2}{R_1} = 1.55$$

Suitable values are R_1 = 2 megohms R_2 = 3.1 megohms.

If a D.C. voltage is required from the slider of the potentiometer at the same level of 1/15 volt/ft. as given by the valve integrator, then since the full travel of the potentiometer represents 900 ft., a 60 volt battery connected across the potentiometer will give the required output from the slider.

The method of putting weathercock stability and rotary damping into the simulator will now be described. We have seen that the physical effect of these forces is to apply torques to the rocket about its centre of gravity which oppose that due to the rudders. Thus in the simulator

/they

they must take the form of voltages fed back into the input of the first integrator which oppose the voltage from the relay contacts. Fig.(iii)(b) shows the first part of the simulator described above modified to include weathercock stability and rotary damping.

It will be seen that the modification consists simply of the addition of the two resistances R_3 and R_4. The resistance R_3 feeds back a current proportional to V_1 and hence to $\frac{d\phi}{dt}$ and thus represents rotary damping.
Similarly R_4 feeds back a current proportional to $\frac{d\Psi}{dt}$ and thus represents weathercock stability. If we now write down the condition that the grid of the first valve remains at constant potential we obtain

$$\frac{V_0}{R_1} + 2C_1\frac{dV_1}{dt} + \frac{V_1}{R_3} + \frac{V_2}{R_4} = 0 \qquad --- \quad 2.10$$

Now if we assume as in equation 2.9 that $V_1 = a_1 \frac{d\phi}{dt}$

$$\text{and } V_2 = a_1 \frac{d\Psi}{dt}$$

Equation 2.10 becomes

$$\pm \frac{a_0 Q}{R_1} - 2a_1 C_1 \frac{d^2\phi}{dt^2} - \frac{a_1}{R_3}\frac{d\phi}{dt} - \frac{a_1}{R_4}\frac{d\Psi}{dt} = 0$$

$$\text{or} \quad \frac{d^2\phi}{dt^2} = \pm \frac{a_0 Q}{2a_1 C_1 R_1} - \frac{1}{2C_1R_3}\frac{d\phi}{dt} - \frac{1}{2C_1R_4}\frac{d\Psi}{dt}$$

Now in equation 2.9 we defined $a_1 = \frac{a_0}{2C_1R_1}$

$$\text{Hence } \frac{d^2\phi}{dt^2} = \pm Q - \frac{1}{2C_1R_3}\frac{d\phi}{dt} - \frac{1}{2C_1R_4}\frac{d\Psi}{dt}$$

Comparing this with equation 1.7c we see that the two are identical if

$$K_3 = \frac{1}{2C_1R_3}$$

$$\text{and } K_2 = \frac{1}{2C_1R_4}$$

Thus we can simulate any required amount of positive weathercock stability or rotary damping by using suitable values for R_4 and R_3. If it is desired to simulate negative weathercock stability the voltage V_2 must be fed back through a valve with unit gain in order to reverse its sign.

3. PROPOSED CONTROL SYSTEM

The control system for Long Shot must satisfy the following requirements:-

(a) The lateral acceleration must not exceed 8g, this limit being fixed by the strength of the wings.

(b) The rocket must return to the line of sight from initial errors up to 150 ft. in as short a time as possible without large overshoot.

/(c)

(c) If the system has a constant lag when the line of sight is moving then this lag must be small compared with the expected lethal range of the rocket, which is about 40 ft.

It is proposed that the information for controlling the rocket will come from two sources. (a) The radio system will provide a measure of h, the lateral misalignment from the line of sight. This information will be sufficiently good to be differentiated once. (b) A linear accelerometer situated near the centre of gravity of the rocket will provide a measure of $\frac{d^2h_o}{dt^2}$, the lateral acceleration of the rocket.

A block schematic of the proposed system is shown in fig. (v).

The voltage V_6 is that obtained from the radio receiver and is assumed to be proportional to the error from the line of sight.

$$\therefore \quad V_6 = a_4 h \qquad 3.1$$

where a_4 is a constant.

The voltage V_2 is obtained from the accelerometer.

$$V_2 = a_2 \frac{d^2 h_o}{dt^2} \qquad 3.2$$

where a_2 is a constant.

Let us assume first that R_4 is small compared with R_1, then the voltage at A, the input of the first amplifier is given by

$$V_A = R_4 \left(\frac{V_6}{R_1} + C_1 \frac{dV_6}{dt} \right)$$

$$V_A = \frac{R_4}{R_1} \left(V_6 + C_1 R_1 \frac{dV_6}{dt} \right)$$

or from 3.1 $V_A = \frac{R_4}{R_1} a_4 \left(h + C_1 R_1 \frac{dh}{dt} \right)$

This voltage is then amplified linearly and limited so that the voltage at B, the output of the limiter, will be given by

$$V_B = \frac{G_1 R_4 a_4}{R_1} \left(h + C_1 R_1 \frac{dh}{dt} \right) \text{ when } |V_E| \leqslant V_L \qquad 3.3a$$

$$\left.\begin{aligned} &\text{or } V_B = + V_L \text{ when } V_E \geqslant V_L \\ &V_B = - V_L \text{ when } V_E \leqslant - V_L \end{aligned}\right\} \quad -- \qquad 3.3b$$

where G_1 is the voltage gain of the amplifier, V_L the limiting voltage and V_E the voltage at the output of the amplifier.

Now if we assume that the input impedance R_5 of the relay amplifier is small compared with R_2 and R_3, the condition that the point D shall be at zero voltage is

$$\frac{V_B}{R_3} + C_2 \frac{dV_2}{dt} + \frac{V_2}{R_2} = 0$$

$$\text{or} \quad \frac{R_2}{R_3} V_B + V_2 + C_2 R_2 \frac{dV_2}{dt} = 0 \qquad 3.4$$

/When

When this condition is satisfied the relay will be on the point of reversing and a very small change in the voltage at D will cause it to open or close.

Substituting from equations 3.3 and 3.2 in 3.4 we obtain for the condition that the voltage at D should be zero.

$$\frac{R_2}{R_3}\frac{G_1R_4A_4}{R_1}\left(h + C_1R_1\frac{dh}{dt}\right) + a_2\left(\frac{d^2h_o}{dt} + C_2R_2\frac{d^3h_o}{dt^3}\right) = o \qquad 3.5a$$

$$\text{when } |V_E| \leqslant V_L$$

$$\pm\frac{R_2}{R_3}V_L + a_2\left(\frac{d^2h_o}{dt^2} + C_2R_2\frac{d^3h_o}{dt^3}\right) = o \text{ when } |V_B| \geqslant V_L \qquad 3.5b$$

For convenience we define the following constants:-

$$\beta = \frac{R_2G_1R_4a_4}{R_3R_1a_2}$$

$$2\alpha = C_1R_1\beta$$

$$\gamma = C_2R_2$$

$$h_c = \frac{1}{\beta}\frac{R_2 V_L}{R_3A_2}.$$

Equations 3.5 then become

$$\left(\beta h + 2\alpha\frac{dh}{dt}\right) + \left(\frac{d^2h_o}{dt^2} + \gamma\frac{d^3h_o}{dt^3}\right) = o \qquad 3.6a$$

$$\text{when } \left|\beta h + 2\alpha\frac{dh}{dt}\right| \leqslant \beta h_c$$

$$\pm\beta h_c + \left(\frac{d^2h_o}{dt^2} + \gamma\frac{d^3h_o}{dt^3}\right) = o \text{ when } \left|\beta h + 2\alpha\frac{dh}{dt}\right| \geqslant \beta h_c \quad 3.6b$$

The quantity h_c may be called the critical error since a steady error of this magnitude is just sufficient to cause the voltage at B to reach the level at which the limiter comes into operation.

The operation of the system can best be understood by considering it in two separate steps. In equations 3.6 (a) and (b) we can consider the first terms with their signs changed namely $-\left(\beta h + 2\alpha\frac{dh}{dt}\right)$ or $\mp\beta h_c$ as functions of the error which demand a certain acceleration in order to satisfy equations 3.6 (a) or (b). We shall consider first the way in which the system responds to a sudden change of acceleration demand. It must be realised that equations 3.6 (a) and (b) are merely conditions for reversal of the relay and are not satisfied at all times. It is clear from equation 1.9 that neither $\frac{d^2h_o}{dt^2}$ nor $\frac{d^3h_o}{dt^3}$ can change instantaneously and hence if the acceleration demand is changed instantaneously then a finite time must elapse before equation 3.6 can again be satisfied. If we examine equation 3.6b it is clear that if the equation remains satisfied for a time large compared with γ, $\frac{d^3h_o}{dt^3}$ will become negligible and we shall have

$$\pm\beta h_c = -\frac{d^2h_o}{dt^2}$$

/Thus

Thus it is clear that if the maximum acceleration allowed by the strength of the wings is $\pm G_o$, then we must arrange that

$$\beta h_c = G_o \qquad \text{--- } 3.7$$

However we must also arrange that $\frac{d^2h_o}{dt^2}$ does not exceed G_o during the periods following a change of acceleration demand when equations 3.6 are not satisfied.

Now in the case of neutral weathercock stability and negligible rotary damping equation 1.9 must always be satisfied,

$$\frac{d^3h_o}{dt^3} + K_1 \frac{d^4h_o}{dt^4} = \pm P \qquad \text{--- } 3.8$$

In order to get a clear picture of the changes of acceleration of the rocket under this type of 'bang-bang' control it is convenient to consider the acceleration $\frac{d^2h_o}{dt^2}$ and the rate of change of acceleration $\frac{d^3h_o}{dt^3}$ as our two variables. The state of the rocket at any given time is completely defined by these two variables. We shall first convert equation 3.8 into an equation in terms of these two variables and then plot its solutions, which will give a geometrical picture of the processes involved.

Let us introduce two non-dimensional variables defined by

$$x = \frac{1}{PK_1} \frac{d^2h_o}{dt^2} \qquad \text{--- } 3.9$$

$$y = \frac{1}{P} \frac{d^3h_o}{dt^3} \qquad \text{--- } 3.10$$

substituting for y in 3.8 we obtain

$$y + K_1 \frac{dy}{dt} = \pm 1 \qquad \text{--- } 3.11$$

or

$$y + K_1 \frac{dy}{dx} \frac{dx}{dt} = \pm 1 \qquad \text{--- } 3.12$$

Now from 3.9

$$\frac{dx}{dt} = \frac{1}{PK_1} \frac{d^3h_o}{dt^3}$$

$$\frac{dx}{dt} = \frac{1}{K_1} y$$

Thus 3.12 becomes

$$y + y \frac{dy}{dx} = \pm 1 \qquad \text{--- } 3.13$$

This equation represents two families of curves in the x y plane, one family representing all possible trajectories when the rudder is hard over in the 'positive' direction and the other all possible trajectories with the rocket hard over in the negative direction.

Equation 3.13 can be integrated on sight, the solutions for the positive and negative signs being

$$x = -y - \log_e (1 - y) + C_1 \qquad \text{--- } 3.14a$$

$$X = -y + \log_e (1 + y) + C_2 \qquad \text{--- } 3.14b$$

The constants C_1 and C_2 are, of course, defined by the value of x at y = o. In fig. (vi) the equations 3.14a and 3.14b are plotted for various values of the arbitary constants C_1 and C_2. We see from

/equation

equation 3.11 that with positive rudder y increases with time and with negative rudder y decreases with time. Hence we see that the movement of the rocket in the xy plane is represented by a point which moves along the appropriate curve in a clockwise direction.

Now suppose that a constant acceleration demand G_4 has been made so that from 3.6 the condition for reversal of the rudders is

$$-G + \frac{d^2h_o}{dt^2} + \gamma\frac{d^3h_o}{dt^3} = 0 \qquad --- \quad 3.15$$

Substituting from 3.9 and 3.10 we obtain

$$PK_1\,x + P\gamma\,y = G$$

$$\text{or}\quad x + \frac{\gamma}{K_1}\,y = \frac{G}{PK_1} \qquad --- \quad 3.16$$

Thus the reversed condition is represented by a straight line in the x y plane with slope $-\frac{K_1}{\gamma}$ which intersects the x-axis at the point $x = \frac{G}{PK_1}$, this line will be called the reversal line. Now it is clear that if the point representing the rocket lies to the left of the reversal line it will move on one of curves of the family represented by 3.14a until it reaches the reversal line, the rudder will then reverse and the representative point will start to move on the curve of the second family which passes through the point of intersection of the first curve with the reversal line. The trajectory in the (x,y) plane after the first reversal depends upon the slope of the reversal line and the slope of the new representative curve. In fig. (vii) four cases have been drawn for a sudden change of acceleration demand from x = - .097 to x = +.097 (this value has been chosen because it represents a change from -8g to + 8g when the constants of the rocket are P = 10500 ft/sec^3 and K = $\frac{1}{4}$ sec.) The reversal line will be defined from 3.16 by

$$x + \frac{\gamma}{K_1}\,y = +.097 \qquad --- \quad 3.17$$

and the trajectories have been drawn for four different values of $\frac{\gamma}{K_1}$. The four lines are BC, BD, BE and BF, having slopes $-\frac{K_1}{\gamma}$ of - 3.0, -4.8, -5.9 and -10.4 respectively. The representative point will start from the point A (-.097, 0) in each case, and will commence to travel clockwise along the curve ACDEF. If the reversal condition is represented by BC then at the point C the rudder reverses and the point would start to move along CK. However the slope of CK at C is greater than the slope of the line BC and hence the point starts to move to the left of the reversal line, which means that the rudder will be reversed again and the process will be repeated. Thus the rudder will oscillate at a theoretically infinite speed to keep the representative point on the line CB until it reaches the point B. In practice the speed of oscillation will be set by the response time and 'backlash' of the relay and rudder system, this will be discussed later. Thus after the point C the equation

$$x + \frac{\gamma y}{K_1} = \frac{G}{PK_1}$$

will always be satisfied and since from the definitions of x and y

$$K_1\frac{dx}{dt} = y$$

the variation of x with time along CB is defined by

$$x + \gamma\,\frac{dx}{dt} = \frac{G}{PK_1}$$

/whose

whose solution is $x = \frac{G}{PK_1} + C_1 e^{-\frac{t}{\gamma}}$ --- 3.18

C_1 is an arbitrary constant whose value will be fixed by the initial conditions at the point C. Thus we see that x approaches its final value exponentially with a time constant γ.

Now let us consider the second reversal condition represented by BD in fig. (vii). Here the slope of the new curve at D is smaller than that of BD and hence the representative point moves over a finite length of curve before intersecting the reversal line again at H. At H oscillations commence as before and the point moves down the line HB.

The third reversal line represents the special case when the new curve actually intersects the reversal line again at the point B. The rudder will then oscillate rapidly to keep the point at B.

The fourth case is represented by BF. By the same arguments as for the other cases we see that the representative point moves over the path AFIJB. It will be observed that the latter part of this path lies to the right of the point B, that is the acceleration 'overshoots' to a larger value than the final one before coming to rest.

In fig. (viii) these four cases have been plotted as graphs of x against t/K_1. It will be seen that as the value of K_1/γ increases the time taken for the acceleration to reach its new value decreases, as we should expect. However if K_1/γ exceeds 5.9 the acceleration overshoots its final value before coming to rest. Thus this value of K_1/γ gives us the fastest change from $x = -.097$ to $x = .097$ without overshoot. The actual time for this case is seen to be $0.9K_1$, and if $K_1 = 0.25$ sec. this time is 0.225 sec.

Thus we see that if we make $\gamma = \frac{K_1}{5.9}$ and make $\beta h_c = 8g$ then the system will obey the first requirement that the lateral acceleration will never exceed 8g.

We can also see that for small changes in acceleration demand the representative point will rapidly reach the reversal line and oscillate along it to the new position. Thus the acceleration will reach its new value exponentially with a time constant γ, where γ is of the order of 1/20th sec.

We can now consider the operation of the control system as a whole. We shall first assume that the time constant of the system is large compared with γ so that we can assume that the motion is represented by equations 3.6 (a) and 3.6 (b) and that the $\gamma \frac{d^3h_o}{dt^3}$ terms can be neglected.

Thus we have

$$\beta h + 2\alpha \frac{dh}{dt} + \frac{d^2h_o}{dt^2} = 0 \qquad \text{--- 3.18 (a)}$$

$$\text{when } \left|\beta h + 2\alpha \frac{dh}{dt}\right| \leqslant \beta h_c$$

$$\pm \beta h_c + \frac{d^2h_o}{dt^2} = 0 \qquad \text{--- 3.18 (b)}$$

$$\text{when } \left|\beta h + 2\alpha \frac{dh}{dt}\right| > \beta h_c$$

We shall now consider the response of the system to an initial error ϵ. The initial conditions are therefore given by $t = 0$, $h_i = 0$, $h = h_o = \epsilon$, $\frac{dh_i}{dt} = \frac{dh}{dt} = \frac{dh_o}{dt} = 0$. We shall assume that the line of sight is stationary so that $h_i = 0$ at all times. Thus $h = h_o$ at all times. From this equation 3.18 becomes

/β h + 2

$$\beta h + 2\alpha \frac{dh}{dt} + \frac{d^2h}{dt^2} = 0 \qquad \text{--- } 3.19\ (a)$$

$$\text{when } \left| \beta h + 2\alpha \frac{dh}{dt} \right| \leqslant \beta h_c$$

$$\pm\beta h_c + \frac{d^2h}{dt^2} = 0 \qquad \text{--- } 3.19\ (b)$$

$$\text{when } \left| \beta h + 2\alpha \frac{dh}{dt} \right| > \beta h_c$$

To get a clear picture of the process described by equations 3.19 we shall use a similar method to that already described. The state of the rocket can be completely defined at a given moment by the quantities h and $\frac{dh}{dt}$ and we shall plot the pair $(h, \frac{dh}{dt})$ as a point in the plane of the variables h, $\frac{dh}{dt}$, which may be called the phase plane. The path of this point gives us a geometrical picture of the process described by equations 3.19.

$$\text{Let} \quad x = h \qquad \text{--- } 3.20\ (a)$$

$$y = \frac{dh}{dt} \qquad \text{--- } 3.20\ (b)$$

First we observe that the divisions between the regions in which 3.19 (a) apply and those in which 3.19 (b) apply are given by

$$\beta x + 2\alpha y = \pm \beta h_c$$

$$\text{or} \quad x + \frac{2\alpha}{\beta} y = \pm h_c \qquad \text{--- } 3.21$$

This represents two parallel straight lines as shown in fig. (ix). The region between these lines, in which equation 3.19 (a) applies may be termed the unsaturated region and the region outside these lines in which 3.19 (b) applies may be termed the saturated region.

Let us assume that the initial error is larger than h_c so that the point at first moves in the saturated region, the movement of the rocket is then defined by:-

$$\pm \beta h_c + \frac{d^2h}{dt^2} = 0$$

$$\text{from } 3.20 \ \frac{d^2h}{dt^2} = \frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$$

$$= y \frac{dy}{dx}$$

$$\therefore \ \pm \beta h_c + y \frac{dy}{dx} = 0 \qquad \text{--- } 3.22$$

The solution is clearly $y^2 = \mp \beta h_c x + A$ which represents two families of parabolas. As before, the representative point will move in a clockwise direction along the appropriate parabola. It is clear that if the initial error is positive and equal to ϵ then the point will move on the parabola

$$y^2 = -\beta h_c (x - \epsilon)$$

The point will continue to move on this parabola until it intersects the line $x + \frac{2\alpha}{\beta} y = + h_c$.

After this the motion will be defined by 3.19 (a)

$$\beta h + 2\alpha \frac{dh}{dt} + \frac{d^2h}{dt^2} = 0$$

/Let

Let us suppose that the motion represented by this equation is not over damped, the necessary condition for this is that

$$\alpha^2 < \beta$$

In this case the solution will be of the form

$$h = A e^{-\alpha t} \cos(w_1 t + \delta)$$

where $w_1 = \sqrt{\beta - \alpha^2}$ and A and δ are arbitary constants.

Thus since $x = h = A e^{-\alpha t} \cos(W_1 t + \delta)$ 3.23 (a)

$$y = \frac{dh}{dt} = -\alpha A e^{-\alpha t} \cos(w_1 t + \delta) - W_1 A e^{-\alpha t} \sin(w_1 t + \delta) \quad 3.23 \text{ (b)}$$

The motion in the (x,y) plane represented by 3.23 (a) and (b) is a spiral which winds round the origin, the representative point moving in a clockwise direction.

Now it is clear that two cases are possible, (a) the spiral may lie completely within the unsaturated region or (b) it may intersect the second boundary line and the representative point will then enter the saturated region again. In the second case the point will move along the appropriate parabola until it again enters the unsaturated region and commences a second spiral section. Thus in general the complete trajectory in the (x,y) plane will consist of a number of sections of parabolas and spirals matched together at the boundary lines. It is clear that the question of whether the trajectory overshoots into the saturated region depends principally upon the size of the initial error and the slope of the boundary line $-\frac{2\alpha}{\beta}$.

There will be less tendency to overshoot if $\frac{2\alpha}{\beta}$ is large. In view of the approximations which have been made in obtaining equations 3.18 (a), a detailed analysis of the conditions for overshoot are scarcely justified. However a numerical example has been plotted in fig. (x), taking values which have been found satisfactory in the simulator. The actual values are $-\frac{2\alpha}{\beta} = 0.4$, $\beta = 16$. With a maximum acceleration of 256 ft/sec^2 (8g) we must make $h_c = 16$ ft. so that $\beta h_c = 256$ ft/sec^2. It will be seen that for an initial error of 150' the representative point remains in the saturated region at all times subsequent to its initial entry, though it is clear that a slightly larger initial error would result in an overshoot. In actual fact there is a small overshoot even for an error of 150' because of the delay in obtaining the required accelerations at the beginning of the trajectory and at the first entry into the unsaturated region. This is shown by fig. (xi) which shows two graphs of error against time. Graph A is drawn from the theoretical curve of fig. (x) and graph B is an actual experimental curve obtained from the simulator with the same values of α and β.

Thus it seems from these considerations that the control system satisfies requirement (b), since the rocket returns to the line of sight from an initial error of 150 ft. in the order of $1\frac{1}{2}$ seconds without large overshoot. It is clear from fig. (ix) that the rocket will return from smaller initial errors in a shorter time with a smaller overshoot.

We must now investigate the response of the system to a continuously moving line of sight. From equation 3.6 (a) we have the reversal condition:-

$$\beta h + 2\alpha \frac{dh}{dt} + \frac{d^2h_o}{dt^2} + \gamma \frac{d^3h_o}{dt^3} = 0$$

and we have shown that after a period of time large compared with γ the rudder will oscillate to keep this condition satisfied.

Now since $h = h_o - h_1$ is the position of the line of sight, this equation may be written

/β(ho

$$\beta (h_o - h_i) + 2\alpha \left(\frac{dh_o}{dt} - \frac{dh_i}{dt}\right) + \frac{d^2h_o}{dt^2} + \gamma\frac{d^3h_o}{dt^3} = o$$

or $$\beta h_o + 2\alpha \frac{dh_o}{dt} + \frac{d^2h_o}{dt^2} + \gamma \frac{d^3h_o}{dt^3} = \beta h_i + 2\alpha \frac{dh_i}{dt} \qquad 3.24$$

The problem is therefore to find the final steady state reached by h_o when h_i is a particular function of time. This steady state will be given by the particular integral of 3.24. The particular integral when h_i is constant is simply $h_o = h_i$, a result which has been tacitly assumed in the discussion of the response to initial errors. The next simplest case is $h_i = A + Bt$, that is h_i has a constant velocity. It can easily be shown that the particular integral of 3.24 is then $h_o = A + Bt$, thus there is no lag for a constant velocity input. However if we now put $h_i = A + Bt + \frac{C}{2} t^2$ we find that the particular integral is

$h_o = A - \frac{C}{\beta} + Bt + \frac{C}{2} t^2$. Thus the rocket lags behind the line of sight

by an amount $\frac{C}{\beta}$, where C is the lateral acceleration of the line of

sight. It is quite clear physically why this occurs. We have already stated that a steady error of magnitude h demands an acceleration of βh, thus if the rocket must accelerate at C ft/sec^2 it can only do so by maintaining a constant error of magnitude C/β. It is also clear that when C reaches the limiting value of 8g the above considerations will no longer apply and the rocket will gradually fall further and further behind the line of sight. We see that if $\beta = 16$, as proposed, the lag in the limit when C = 8g is equal to 16 ft. It can be shown that this acceleration is not exceeded for reasonable target manoeuvres. Thus we see that the system also satisfies requirement (b).

Results obtained with simulator and Proposed Control System

The simulator has been used mainly to determine the most suitable values for the various constants. Fig. (xii) shows the response to initial errors of 50 ft., 100 ft. and 150 ft. with $\frac{2\alpha}{\beta} = 0.44$ and $\beta = 16$.

Fig. (xiii) shows the response to initial errors of 150 ft. with varying amounts of positive weathercock stability. From equation 1.11 we see that when K_3 can be neglected, the final steady acceleration reached by the rocket when the rudder is held over one way is given by P/K_2. Thus if $P = 10500$ ft./sec^3 and $K_2 = 33$ sec^{-1} the final acceleration is limited to 320 ft/sec^2 or 10g. It was found that with values of K_2 of less than 33 sec^{-1} there was no measurable difference in the performance of the system compared with its performance with $K_2 = o$.

As K_2 is increased further the acceleration which the rocket can attain becomes limited below 10g and consequently the response of the whole system becomes slower and tends to overshoot rather more. This is shown by the graphs in fig. (xiii). We thus conclude that values of K_2 from o to 40 sec^{-1} can be tolerated and the system does not become unstable for even larger values to K_2.

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Author: J. Rutherglen.

Approved: R. Cockburn.

APPENDIX I List of Symbols used in describing the aerodynamic characteristics of Long Shot.

SYMBOL	DESCRIPTION	DIMENSIONS
Ψ	Angle between tangent to flight path and fixed datum line	Radians
ϕ	Angle between heading and fixed datum line	Radians
α	Angle of incidence or yaw, equal to $(\phi - \Psi)$	Radians
h_1	Lateral distance of rocket from fixed datum	Feet
h_o	Lateral distance of line of sight from fixed datum	Feet
h	Lateral distance of rocket from line of sight, equal to $h_o - h_1$	Feet
V	Forward velocity of rocket	Feet/sec.
K_1	Constant of proportionality between angle of incidence and rate of turn of flight path	Time
Q	Initial angular acceleration of heading produced when full rudder is applied	Radians/sec^2
P	Final rate of change of lateral acceleration produced when full rudder is applied with neutral weathercock stability, equal to QV	Feet/sec^3
K_2	Weathercock stability constant defined as reduction of angular acceleration of heading per unit rate of turn of flight path	Sec^{-1}
K_3	Rotary damping constant expressed as reduction of angular acceleration of heading per unit rate of turn of heading	Sec^{-1}

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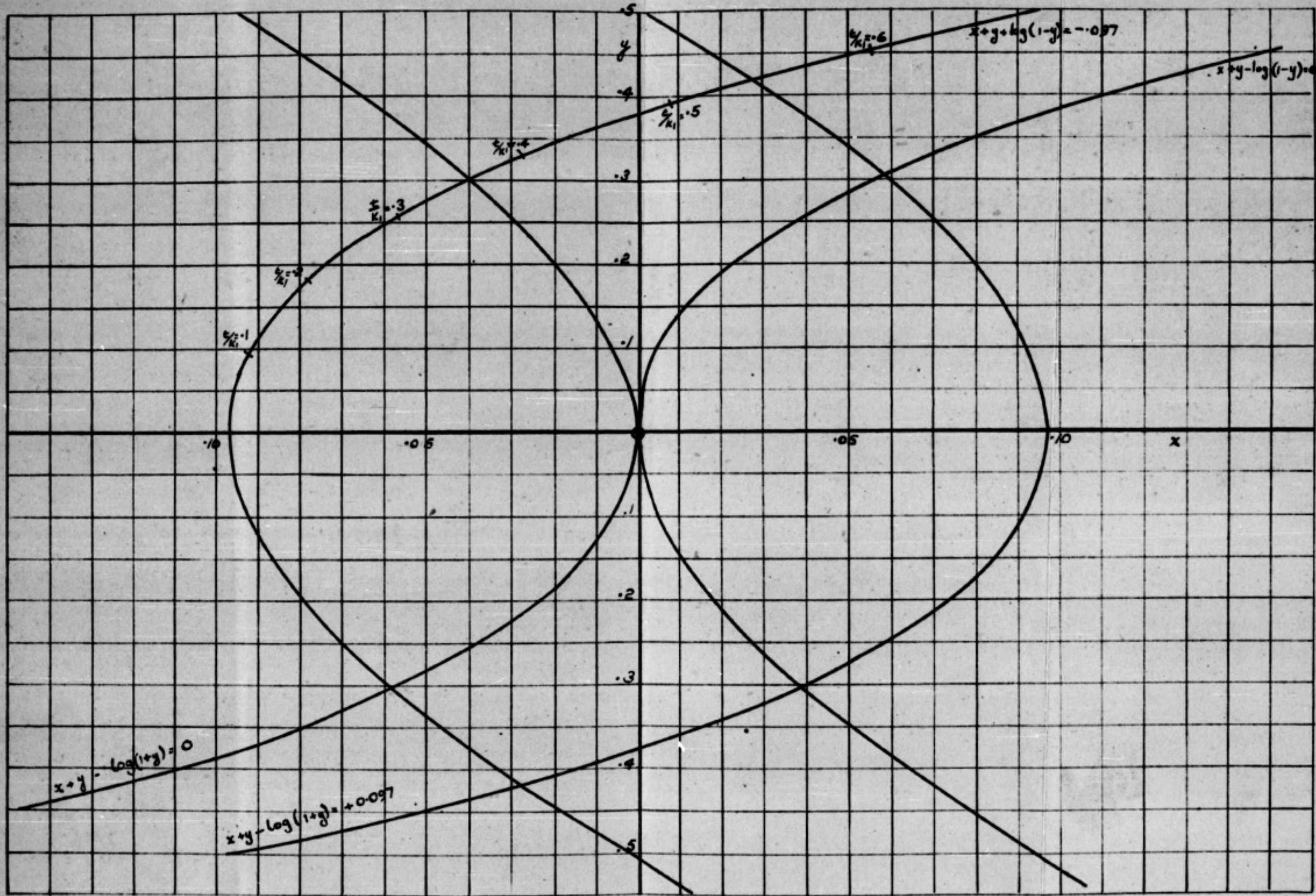

FIG. VI

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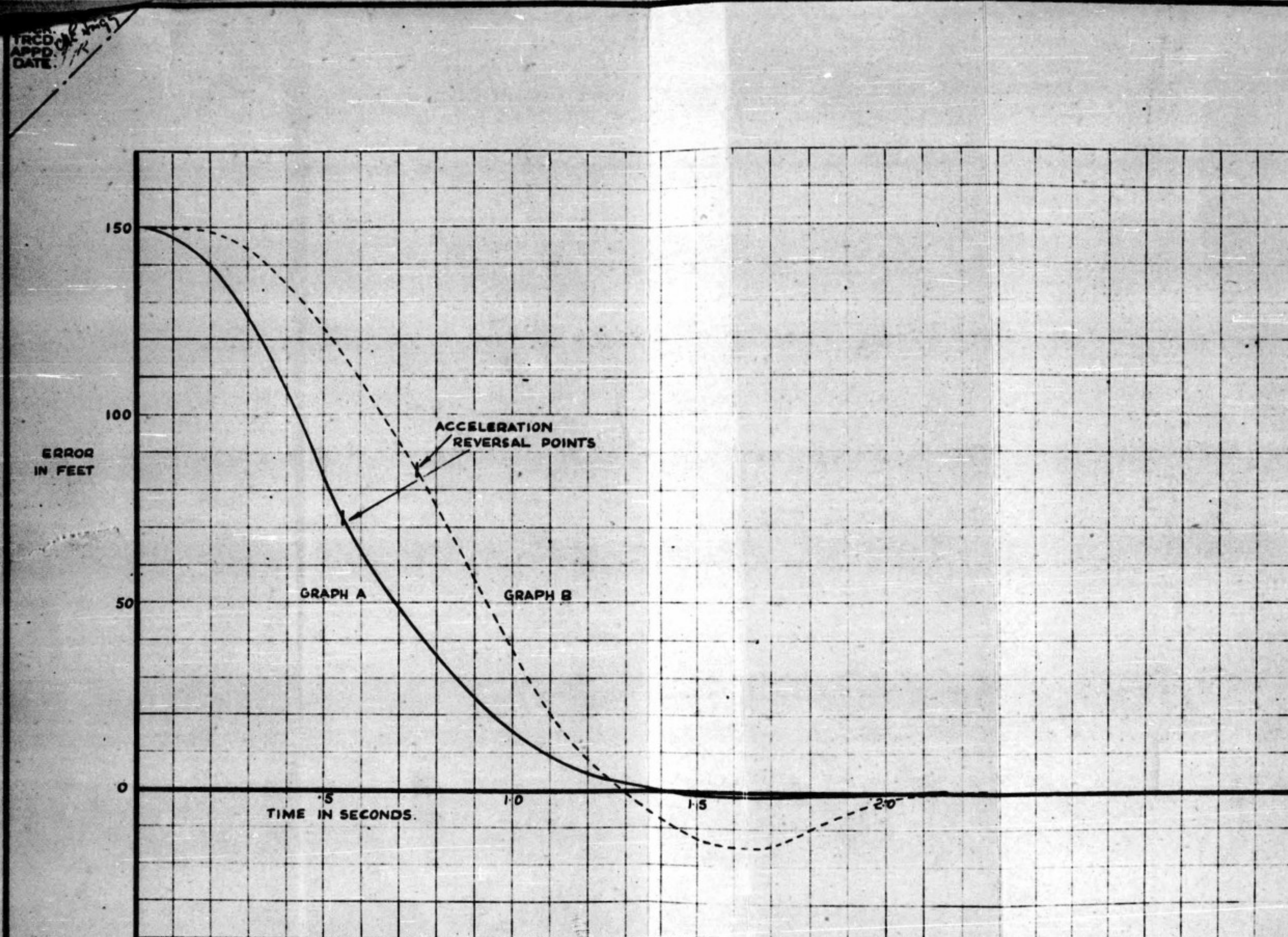

GRAPH A — THEORETICAL RESPONSE OF SYSTEM DEFINED BY EQUATIONS 3·18 (a) & (b) WITH $\frac{2d}{\beta} = 0{\cdot}4$, $\beta = 16$, ACCELERATION LIMITED TO 8g

GRAPH B — MEASURED RESPONSE OF SIMILAR SYSTEM ON SIMULATOR WITH ROCKET CONSTANTS $p = 10500$ FT/SEC^3, $K_1 = 1/4$ SEC, $K_2 = K_3 = 0$.

RESPONSE OF CONTROL SYSTEM TO INITIAL ERROR OF 150 FEET.

FIG. (Xi) 5

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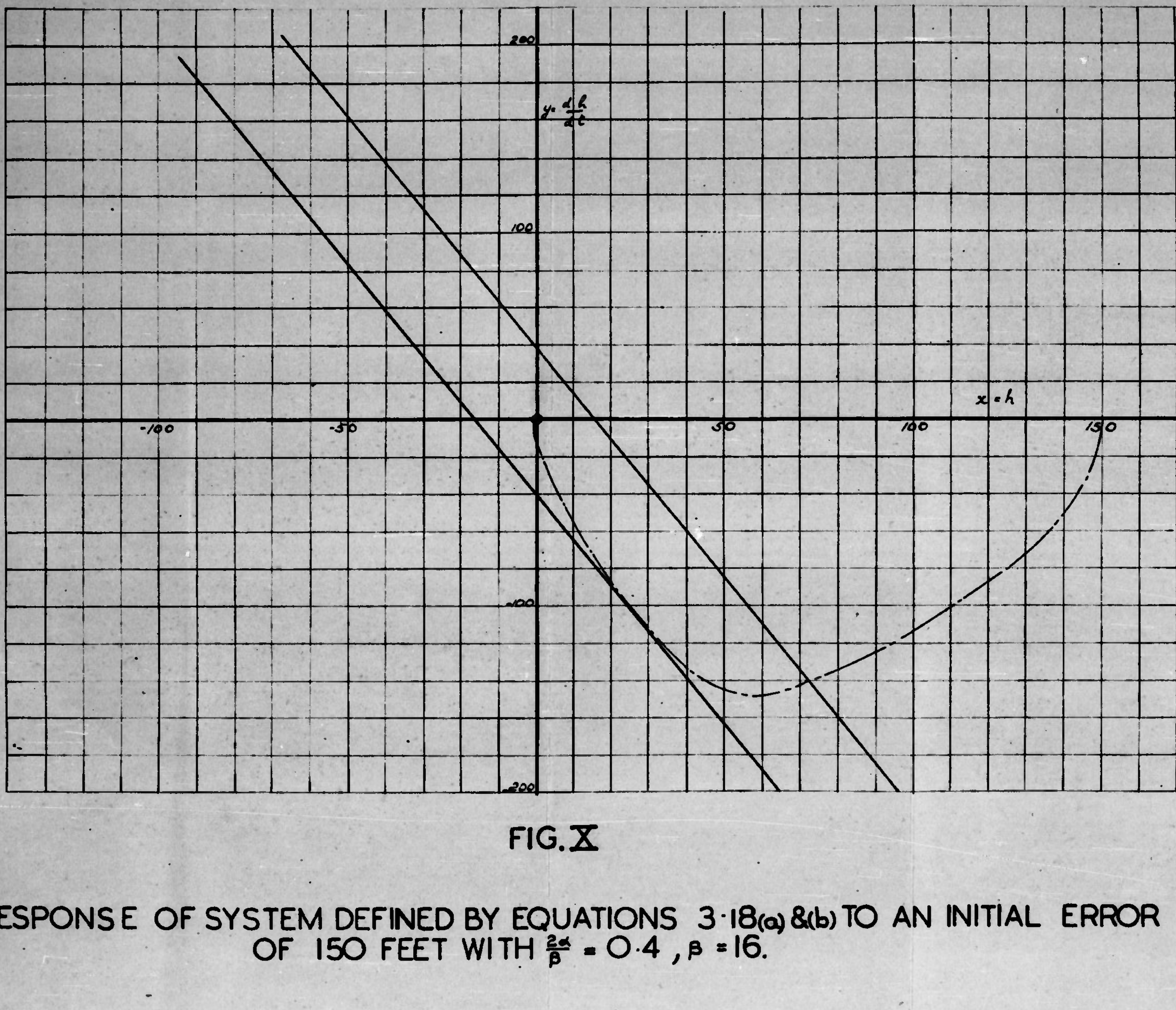

FIG. X

RESPONSE OF SYSTEM DEFINED BY EQUATIONS 3·18(a) &(b) TO AN INITIAL ERROR OF 150 FEET WITH $\frac{2\alpha}{\beta} = 0{\cdot}4$, $\beta = 16$.

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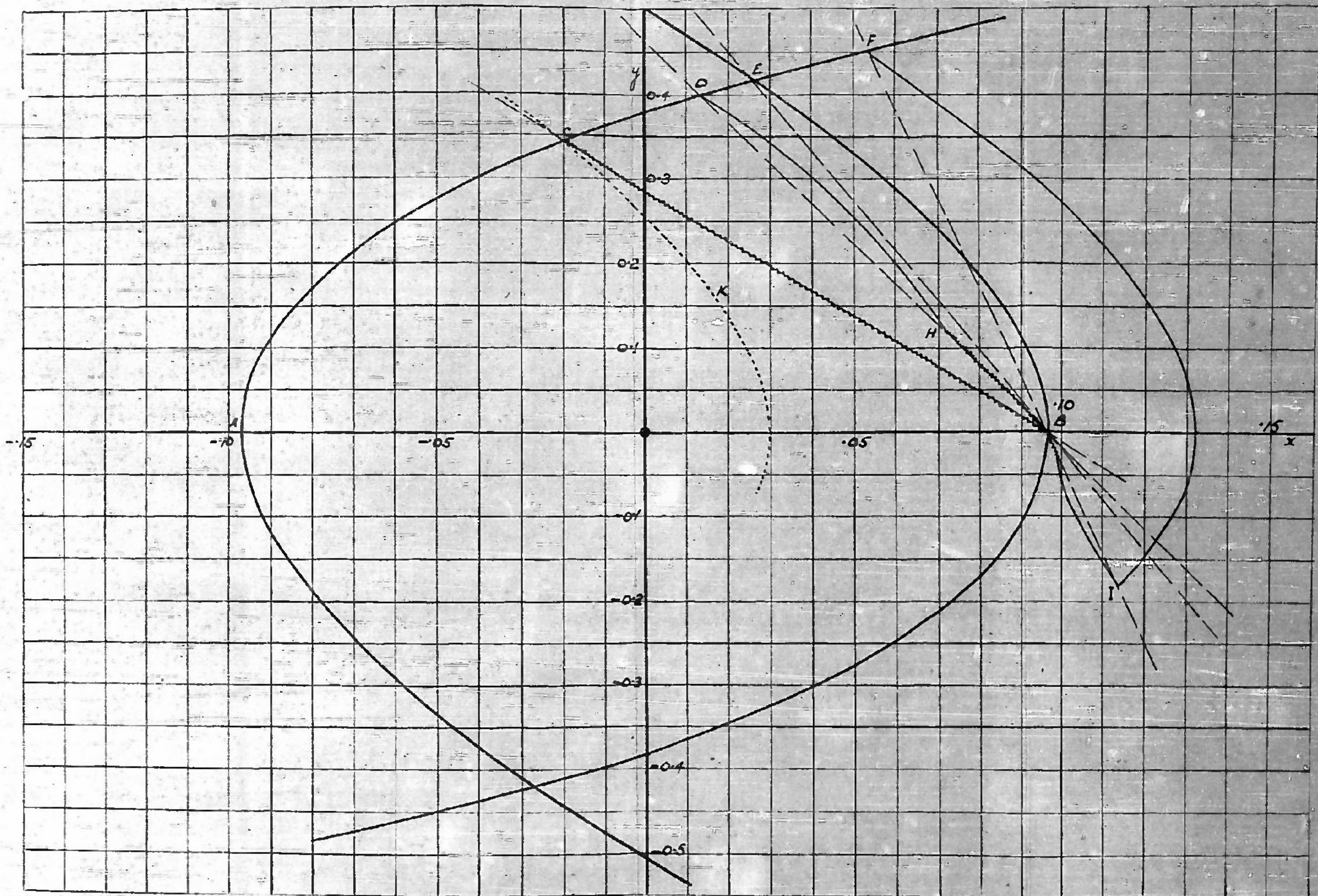

(FIG VII)

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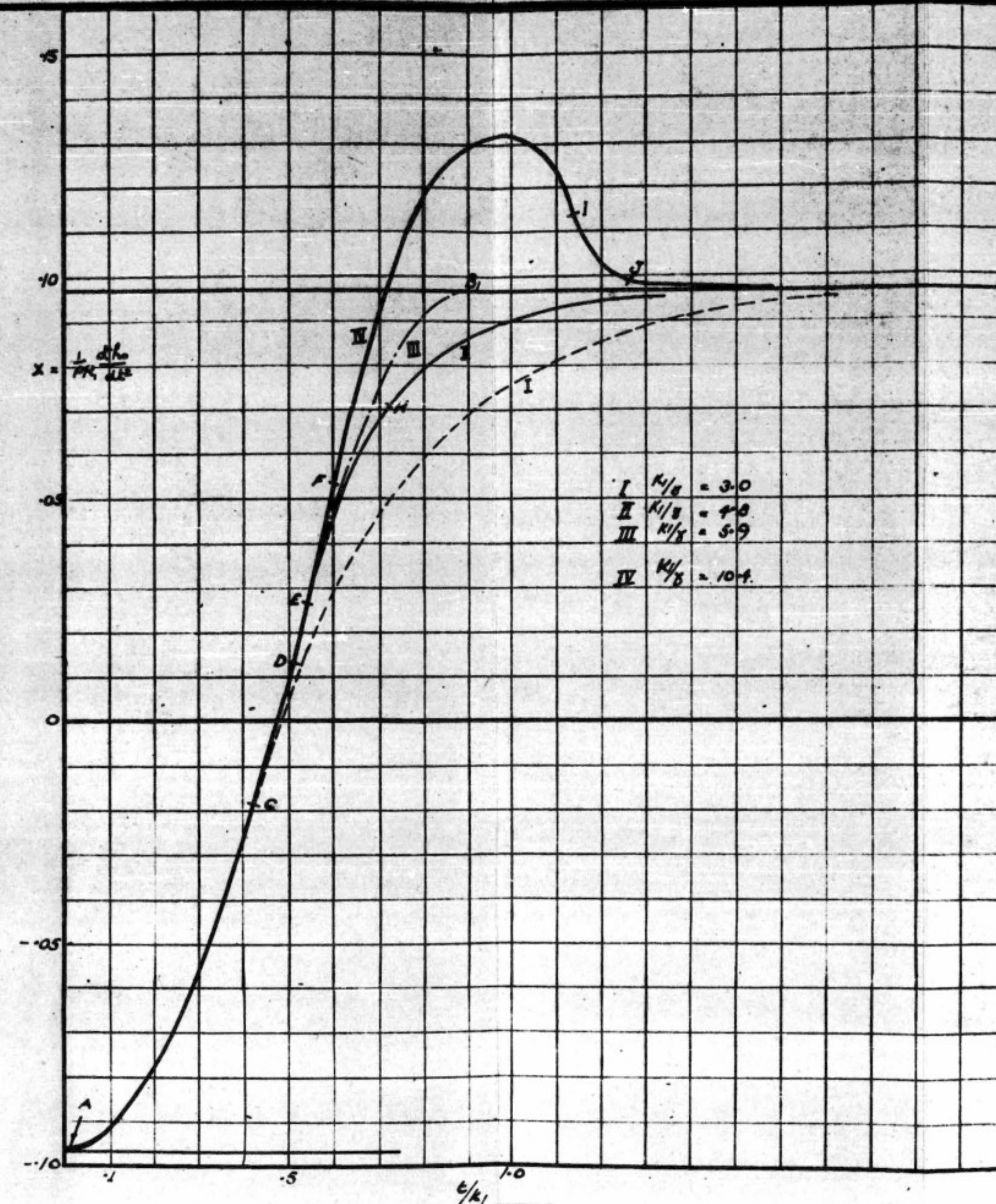

FIG. VIII

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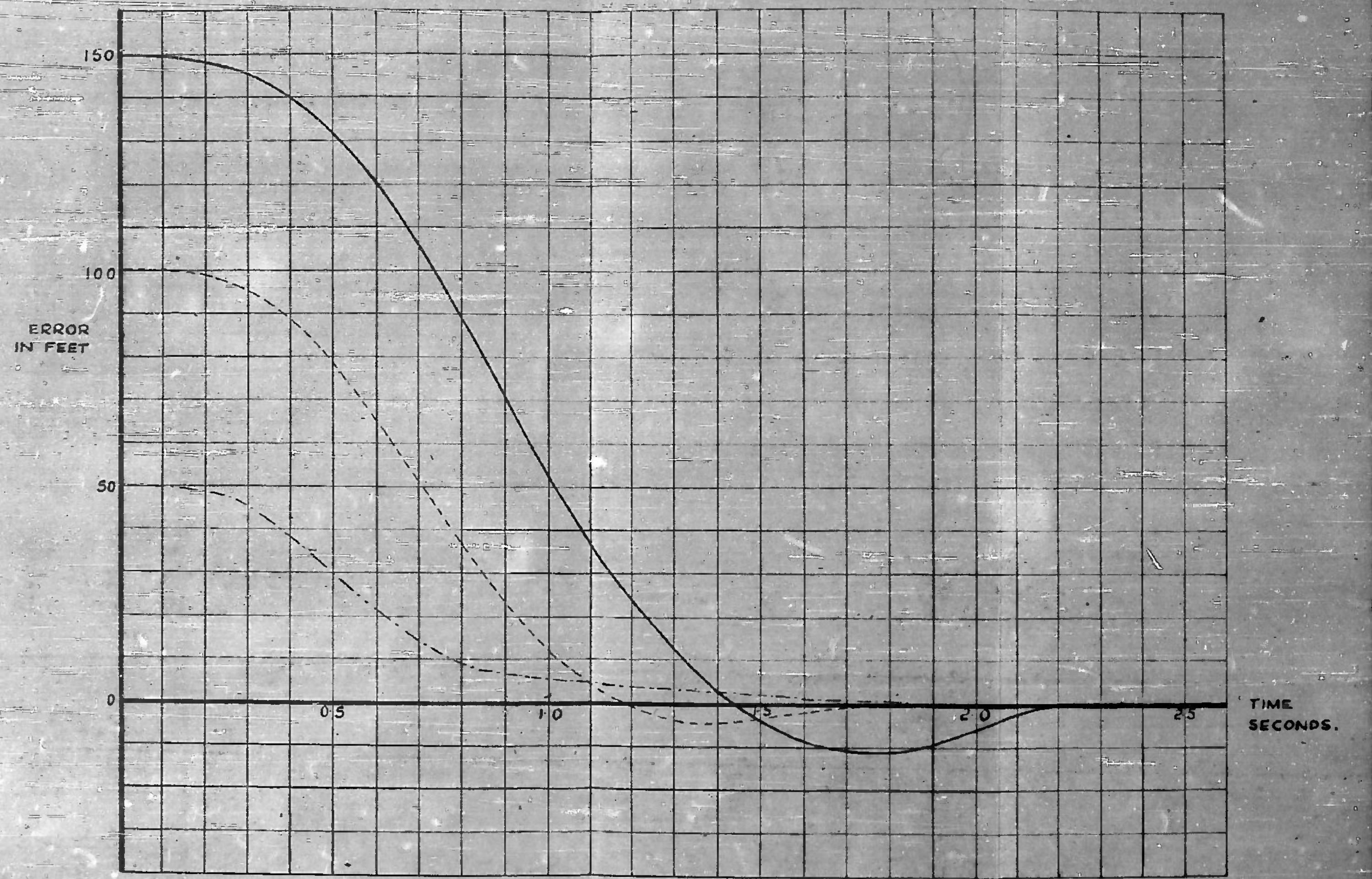

ROCKET CONSTANTS. $P = 10500\ ft/sec^3$ $K_1 = 1/4\ sec.$ $K_3 = 0$
NEUTRAL WEATHERCOCK STABILITY ($K_2 = 0$)
CONTROL CONSTANTS. ACCELERATION LIMITED TO $8g$

$\frac{2d}{\beta} = 0{\cdot}44$ $\beta = 16$

SIMULATOR RESPONSE TO INITIAL ERRORS.

FIG (XII)

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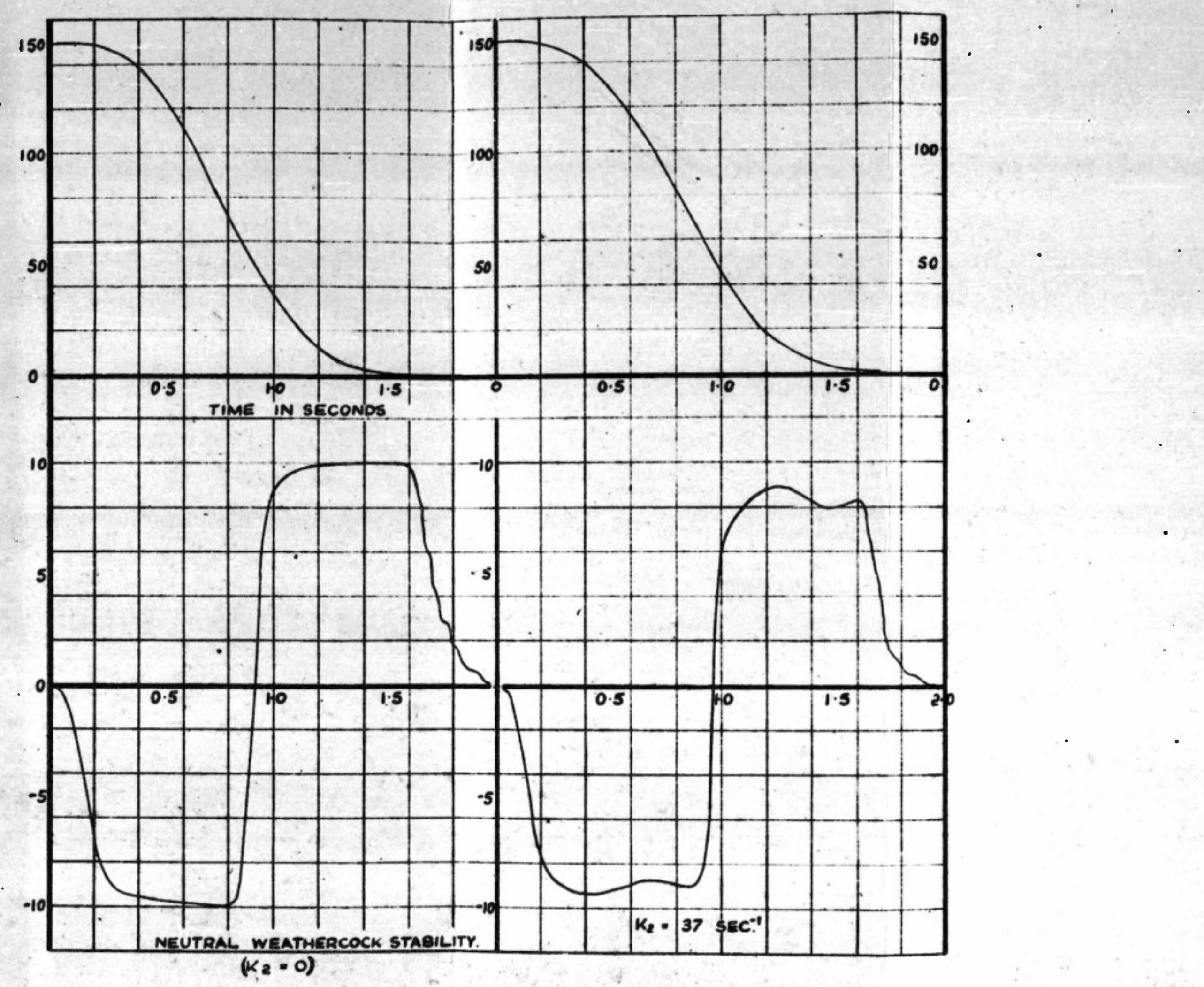

ROCKET CONSTANTS $P = 10500$ ft/sec^3 $K_1 = 1/4$ sec $K_3 = 0$
CONTROL CONSTANTS ACCELERATION LIMITED TO 10g
$\frac{2d}{\beta} = \cdot 44$ $\beta = 16$

SIMULATOR RESPONSES TO INITIAL ERROR OF 150 FEET WITH VARYING AMOUNTS OF WEATHERCOCK STABILITY.

FIG. (XIII)a.

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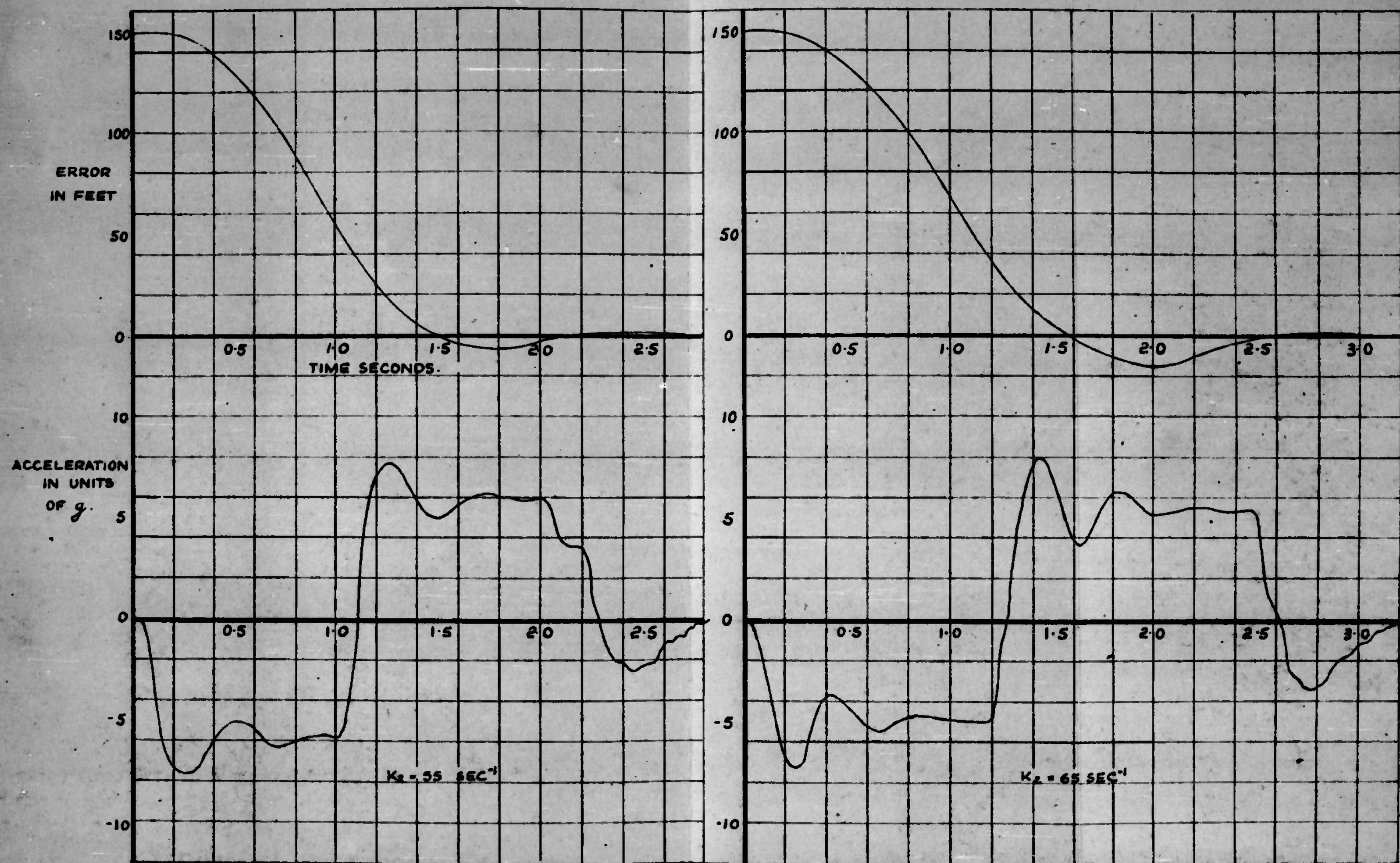

ROCKET CONSTANTS $P = 10500\ ft/sec^3$. $K_1 = 1/4\ sec$ $K_3 = 0$

CONTROL CONSTANTS ACCELERATION LIMITED TO 10g.

$\frac{2d}{\beta} = \cdot 44$ $\beta = 16$

SIMULATOR RESPONSES TO INITIAL ERROR OF 150 FEET WITH VARYING AMOUNTS OF WEATHERCOCK STABILITY.

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FIG.(XIII)b

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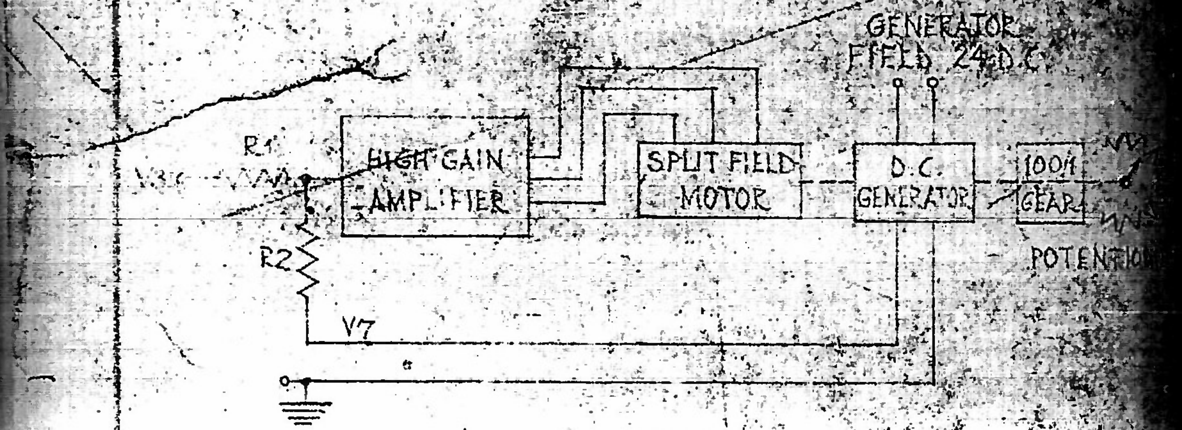

FIG. (iv)

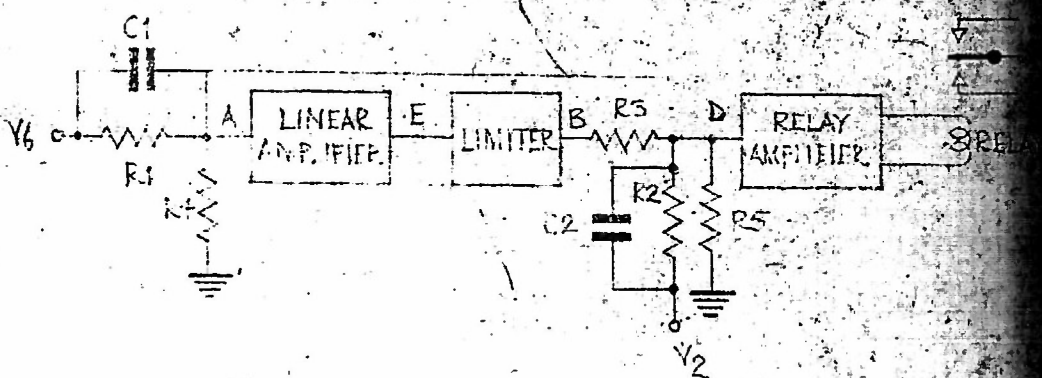

FIG (v)

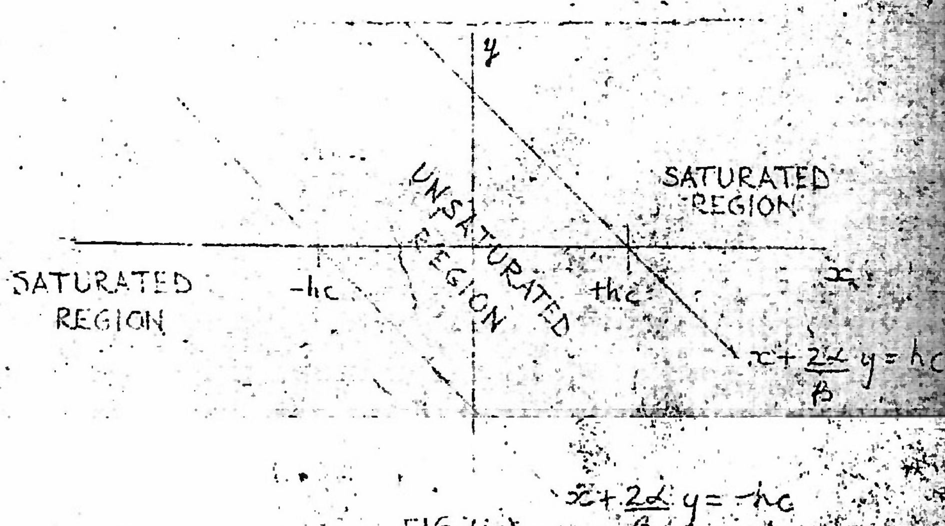

FIG. (ix)

LONG SHOT REPORT FIGS (iv, v & ix)

ISSUE No 1

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FRAME

1006

SECRET (OVER)

TITLE: Electrical Simulator and Proposed Control System for Long Shot

AUTHOR(S): Rutherglen, J.
ORIGINATING AGENCY: Telecommunications Research Establishment, Malvern
PUBLISHED BY: (Same)

ATI- 1003
DIVISION (None)
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DATE	DOC. CLASS.	COUNTRY	LANGUAGE	PAGES	ILLUSTRATIONS
May '45	Secr.	Gt. Brit.	Eng.	26	diagrs, graphs

ABSTRACT:

The rocket Longshot is launched from a fighter, is automatically guided toward an enemy aircraft and employs a control system designed for a line of sight trajectory in which it flies on a radio beam kept pointed at the target. To investigate the stability and performance of various control systems, an electrical simulator was constructed which produces voltages proportional to the movement of the rocket after the application of the controlling rudders. The aerodynamic characteristics of the rocket and the way in which they are simulated are described. A diagram shows a proposed control system which gave satisfactory performance on the simulator.

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DIVISION: Guided Missiles (1)
SECTION: Design and Description (12)

ATI SHEET NO.: S-1-12-107

SUBJECT HEADINGS: Missiles, Guided - Foreign research (62935); Missiles, Guided - Control systems - Testing (62808); Missiles, Guided - Aerodynamics (62350); Simulators, Control (86725)

CLASSIFICATION CHANGED

TO SECRET

AUTH Navy Dept. Bulletin NAVEXOS, P491 dtd Jan-June 1947.

BY George R. Jordan, USCO

DATE ####

Distribution Change per H. R.

SECRET

TITLE: Electrical Simulator and Proposed Control System for Long Shot

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May '45	Secr.	Gt. Brit.	Eng.	28	diagrs, graphs

ABSTRACT:

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